

Mathematically Combining Car Shapes

Paul Mayer, June 22, 2014

I have read articles on the mathematically averaged features of human faces. The thesis of these studies is that the more faces that are added to the averaging calculation, the more attractive the face. There have also been counter arguments by others that individual attractive faces are more appealing than mathematically averaged faces. It got me thinking, what about applying that same technique to averaging car shapes? So, for example, a car designer could ask the computer to combine a Porsche, a Ferrari, and a McLaren F1, and Dodge Viper, let's say 25% Porsche 911 Coupe, 25% Ferrari 348 TS, 25% McLaren F1, and 25% Dodge Viper. This analysis will be worked out to a small extent below. Other questions that one can be ask. What is the average looking American sedan by model year? How does that average shape evolve over the years? Certainly we know that cars from each era have a characteristic look. A mathematical approach could make that design evolution very precise. What is the one standard deviation from the average sedan look like by model year, the two standard deviations car by model year? Is there a correlation between popularity of a car design (as measured by sales volume or popularity among collectors) and standard deviation from the average?

An example of combining two car shapes is shown in Figure 1. It is a three quarter view of a combination of 40% 1997 Ferrari F355 Berlinetta and 60% 1967 Alfa Romeo Giulia Sprint GT Veloce. I created this image with morphing software years ago when I first thought of this idea. The car color is purple because the original images were red and blue respectively. Ultimately one needs to scan 3 dimensional coordinates of the entire surface of each car to do full justice to this idea.



Figure 1. 40% 1997 Ferrari F355 Berlinetta and 60% 1967 Alfa Romeo Giulia Sprint GT Veloce

Using the morphing software FantaMorph, I came up with some examples of combining multiple car shapes. Below I work out a morph (average) of four cars in profile. I like the FantaMorph software, easy to use, and at \$30 for the basic version, I think a good deal. I tried some freeware morphing software, but the quality of the results was not as good. These images are from the book Cars Visual Encyclopedia, pages 323, 341, 343, 435. Hopefully I can claim fair use.



Figure 2. McLaren F1 side profile

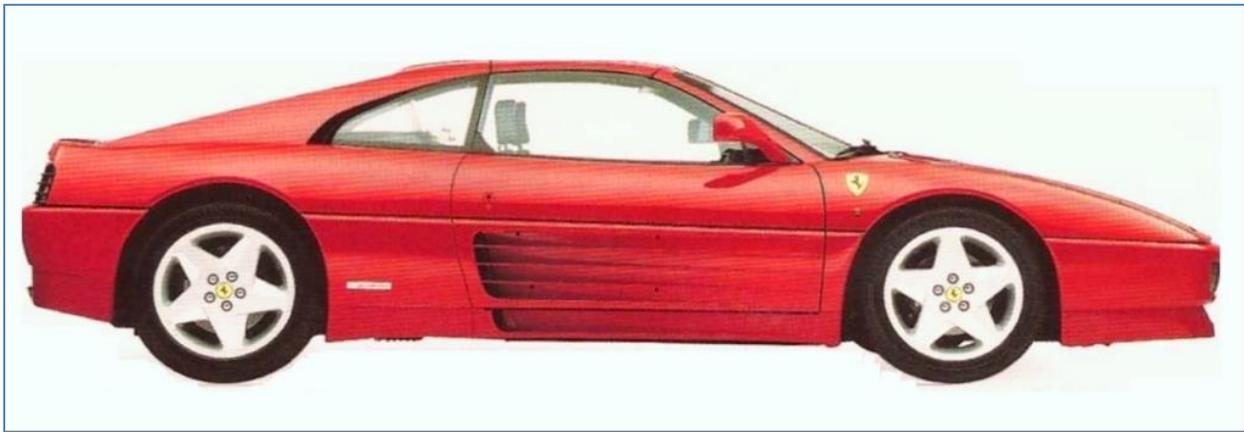


Figure 3. Ferrari 348 TS side profile



Figure 4. 50% Ferrari 348 TS and 50% McLaren F1

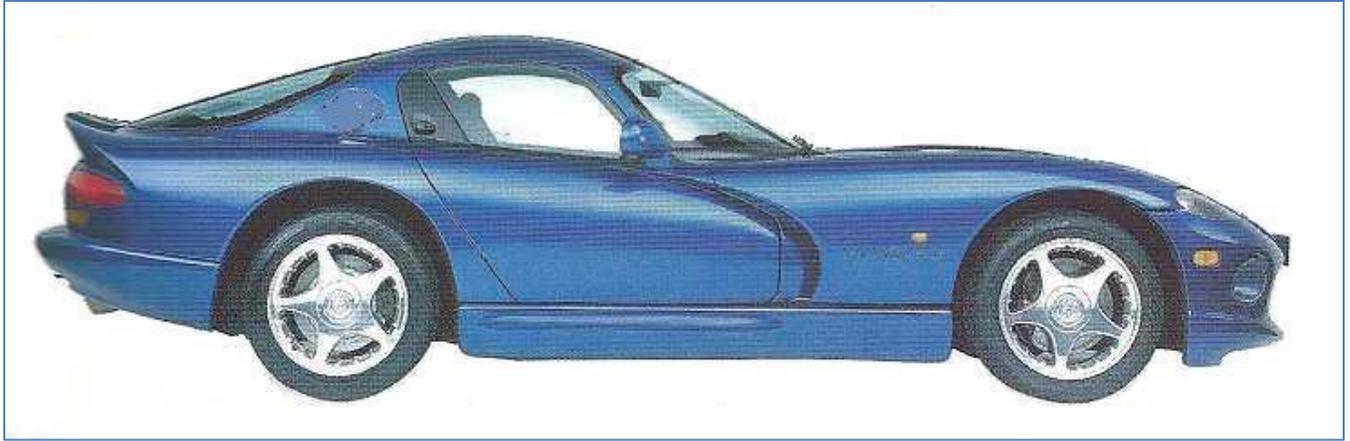


Figure 5. Dodge Viper side profile

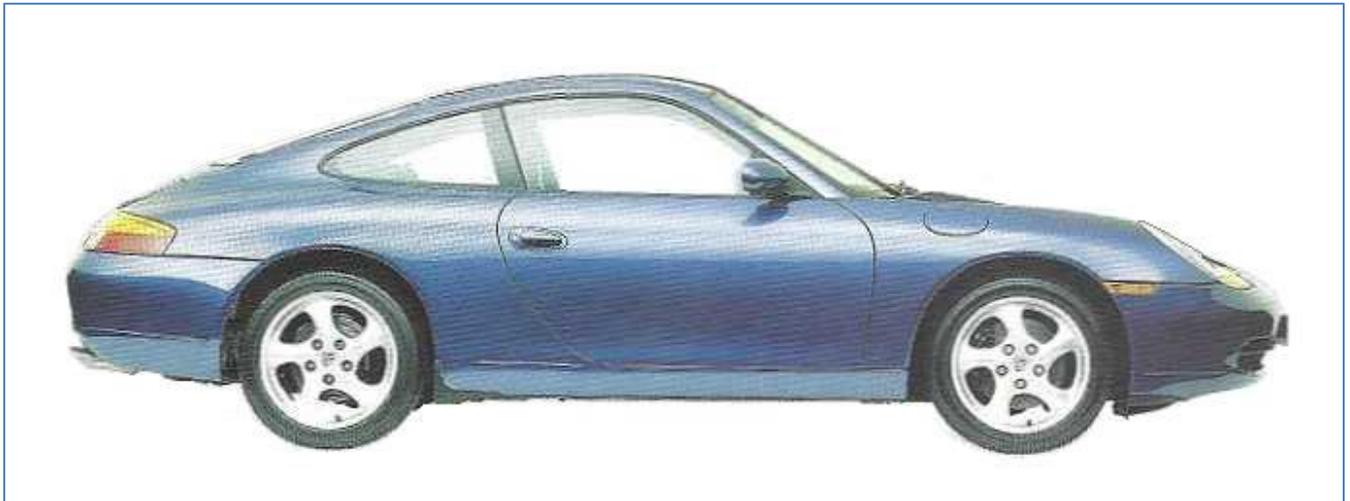


Figure 6. Porsche 911 Coupe side profile

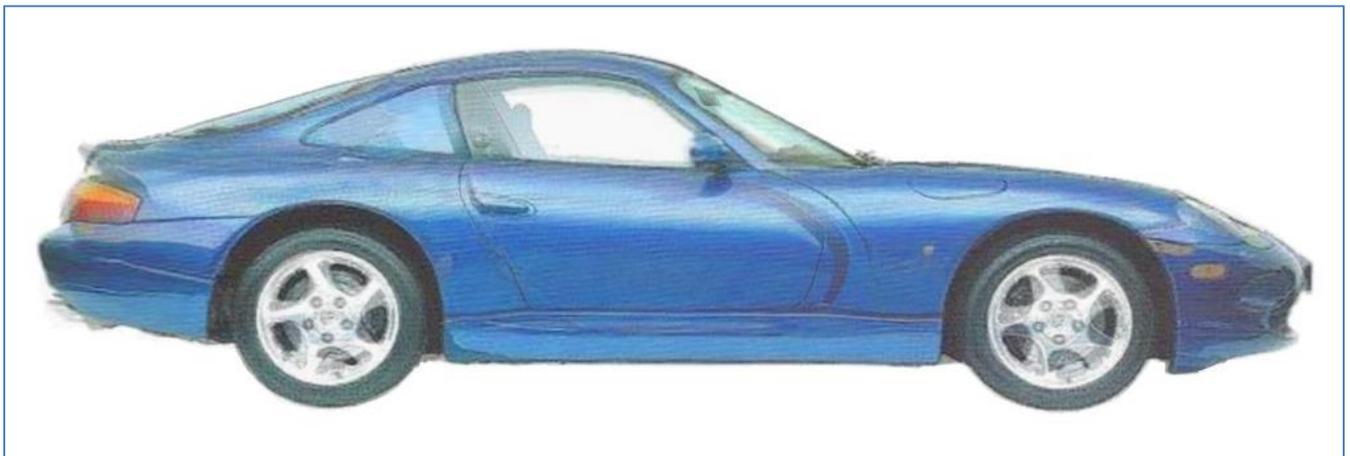


Figure 7. 50% Porsche 911 Coupe and 50% Dodge Viper

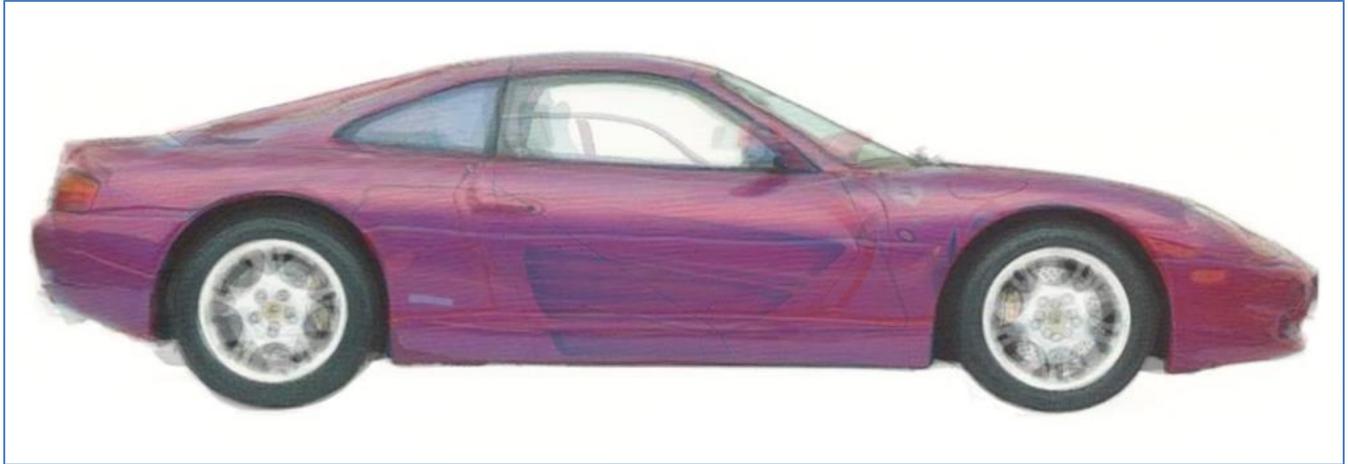


Figure 8. 25% Ferrari 348 TS, 25% McLaren F1, 25% Porsche 911 Coupe, 25% Dodge Viper

As mentioned earlier one needs to scan 3 dimensional coordinates of the entire surface of each car to do full justice to this idea. Budget considerations keep this 2 dimensional for now. I will measure coordinates of some key points on the four profiles shown above and calculate averages and standard deviations. One question to consider, should I 'normalize' the car shapes by scaling them so that each wheelbase is identical? I think that would make for a more fair comparison between car shapes. To get an idea of the mathematics, I selected 13 points 'A' to 'M' on each profile. I inserted each image into a CAD package (free one called Draft Sight). I determined the X and Y coordinates of each point.

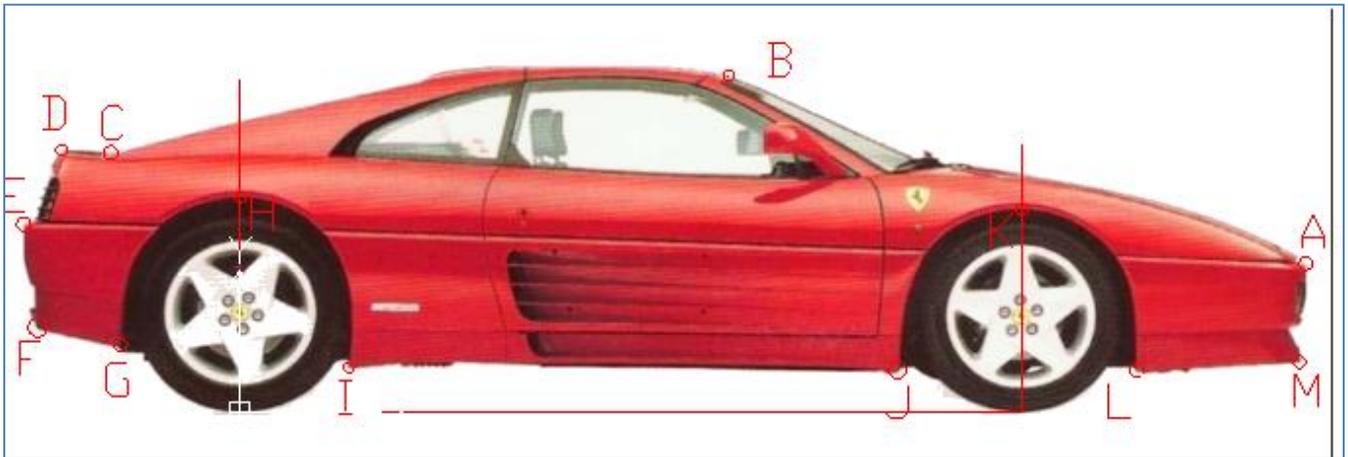


Figure 9. Ferrari side profile with measurement points shown

The coordinate points below are normalized by dividing each value by the wheelbase dimension. The origin is defined as the bottom most point of the rear tire.

MCLAREN F1			PORSCHÉ 911			VIPER			FERRARI TS		
POINT	X	Y	POINT	X	Y	POINT	X	Y	POINT	X	Y
A	1.29	0.13	A	1.37	0.24	A	1.29	0.20	A	1.36	0.19
B	0.67	0.38	B	0.56	0.52	B	0.41	0.47	B	0.63	0.43
C	0.06	0.32	C	-0.18	0.41	C	-0.29	0.38	C	-0.17	0.33
D	-0.16	0.31	D	-0.30	0.35	D	-0.38	0.39	D	-0.23	0.33
E	-0.19	0.20	E	-0.38	0.25	E	-0.42	0.23	E	-0.28	0.24
F	-0.15	0.09	F	-0.34	0.14	F	-0.41	0.20	F	-0.26	0.11
G	-0.14	0.08	G	-0.15	0.10	G	-0.18	0.10	G	-0.15	0.09
H	0.00	0.26	H	0.00	0.29	H	0.00	0.30	H	0.00	0.27
I	0.15	0.05	I	0.16	0.07	I	0.17	0.07	I	0.14	0.06
J	0.84	0.05	J	0.84	0.08	J	0.83	0.06	J	0.84	0.05
K	0.99	0.24	K	1.00	0.29	K	1.00	0.29	K	1.00	0.26
L	1.12	0.05	L	1.15	0.06	L	1.16	0.05	L	1.15	0.05
M	1.27	0.07	M	1.34	0.11	M	1.29	0.09	M	1.36	0.06

AVERAGE			STDEV		
POINT	X	Y	POINT	X	Y
A	1.33	0.19	A	0.045	0.047
B	0.57	0.45	B	0.114	0.062
C	-0.14	0.36	C	0.145	0.044
D	-0.27	0.35	D	0.092	0.036
E	-0.31	0.23	E	0.104	0.021
F	-0.29	0.13	F	0.108	0.047
G	-0.15	0.09	G	0.016	0.008
H	0.00	0.28	H	0.001	0.019
I	0.15	0.06	I	0.013	0.011
J	0.84	0.06	J	0.004	0.014
K	1.00	0.27	K	0.004	0.023
L	1.15	0.05	L	0.016	0.005
M	1.31	0.08	M	0.041	0.020

Figure 10. Data for measurement points, normalized to wheelbase = 1

The tables below are taking the model specific coordinate value minus the average, divided the standard deviation for that point. ABS DIFF = ABS((X - AvgX)/StdevX).

ABS DIFF FROM AVG FERRARI TS			ABS DIFF FROM AVG MCLAREN F1			ABS DIFF FROM AVG PORSCHÉ 911			ABS DIFF FROM AVG VIPER		
	X	Y		X	Y		X	Y		X	Y
A	0.77	0.01	A	0.91	1.35	A	0.95	1.04	A	0.82	0.30
B	0.53	0.33	B	0.88	1.16	B	0.01	1.22	B	1.39	0.26
C	0.15	0.68	C	1.38	0.97	C	0.22	1.16	C	1.01	0.49
D	0.43	0.36	D	1.13	1.06	D	0.38	0.09	D	1.18	1.32
E	0.36	0.38	E	1.23	1.29	E	0.59	1.08	E	1.00	0.16
F	0.28	0.54	F	1.25	0.93	F	0.44	0.12	F	1.09	1.35
G	0.12	0.77	G	1.02	0.95	G	0.23	0.98	G	1.37	0.74
H	1.41	0.38	H	0.03	1.18	H	0.54	0.42	H	0.85	1.14
I	1.16	0.37	I	0.33	1.24	I	0.26	0.73	I	1.22	0.88
J	1.01	0.46	J	0.59	0.86	J	0.37	1.43	J	1.22	0.11
K	0.50	0.43	K	1.50	1.21	K	0.50	0.71	K	0.50	0.93
L	0.07	0.12	L	1.43	1.29	L	0.54	1.15	L	0.81	0.01
M	0.99	0.91	M	1.07	0.68	M	0.70	1.27	M	0.62	0.32
AVG	0.60	0.44	AVG	0.98	1.09	AVG	0.44	0.88	AVG	1.01	0.62
X & Y COMBINED = (0.60^2 + 0.44^2)^0.5 = 0.74			X & Y COMBINED 1.46			X & Y COMBINED 0.98			X & Y COMBINED 1.18		

Figure 11. Statistics on the data points

The final number is a measure of the distance from average for each car. We can see that the McLaren is the most 'unlike' the average car shape, and the Ferrari TS the most 'like' the average car shape.